DOCUMENT RESUME

ED 423 841 IR 019 062

AUTHOR	Johari, Abbas
TITLE	Effects of Inductive Multimedia Programs Including Graphs on Creation of Linear Function and Variable Conceptualization.
PUB DATE	1998-02-00
NOTE	15p.; In: Proceedings of Selected Research and Development Presentations at the National Convention of the Association for Educational Communications and Technology (AECT) Sponsored by the Research and Theory Division (20th, St. Louis, MO, February 18-22, 1998); see IR 019 040.
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Academic Achievement; *Computer Assisted Instruction; Computer Literacy; Concept Formation; Courseware; Educational Strategies; Functions (Mathematics); Graphs; Higher Education; Induction; Instructional Design; *Instructional Effectiveness; Learner Controlled Instruction; *Mathematical Concepts; *Mathematics Instruction; Multimedia Instruction; *Multimedia Materials; Pretests Posttests; Problem Solving; Research Needs; Tables (Data); Undergraduate Study; *Visual Aids; Word Problems (Mathematics)

ABSTRACT

This study examined the effects of an inductive multimedia program, including graphs, on subjects' ability to create linear functions and conceptualize variables from word problems. Subjects were 98 undergraduate students in two sections of a computer literacy course. Students' achievements were assessed via pre- and posttests, which were parallel to the instructional themes stated in the treatment programs. Students were randomly assigned to one of two treatment groups to view a version of a self-paced program. Students, regardless of treatment, scored significantly higher on posttest than pretest on both function construction and variable conceptualization. These results may have been influenced by instructional strategies, including: inquiry mathematical thinking, schema training, linked representational systems, and coordinate graph tutorial teaching. Students receiving instruction via the inductive table-and-graph program scored significantly higher on the function construction of the posttest than did students receiving the table-only treatment. Results suggest the use of inductive multimedia program treatments that incorporate many strategies including inquiry learning from data, tutorial, schema, and core representational systems for the problem of translation, specifically creation of linear function. Data specifically suggest that inductive multimedia programs that include the coordinate graph tutorial strategy have a significant effect on the function construction tasks. (Contains 101 references.) (Author/AEF)

Effects of Inductive Multimedia Programs Including Graphs on Creation of Linear Function and Variable Conceptualization

By:
Abbas Johari
Arizona State University

Effects of Inductive Multimedia Programs Including Graphs on Creation of Linear Function and Variable Conceptualization

Abbas Johari
Arizona State University

Abstract

This study examined the effects of an inductive multimedia program, including graphs, on subjects' ability to create linear functions and conceptualize variables from word problems. The subjects were 98 undergraduate students, enrolled in two sections of a computer literacy course at a large southwestern university. Students' achievements were assessed via 12-item, short answer, pre- and posttests, which were parallel to the instructional themes stated in the treatment programs.

Students were randomly assigned to one of two treatment groups to view a version of a self-paced program. Treatment 1, the inductive table-only program, included tables in addition to other multimedia elements and was developed based on many instructional strategies including inquiry inductive learning strategy. Treatment 2, the inductive table-and-graph program, included both tables and graphs and was identical to the first version in all respects except for the addition of the graph visuals and graph-related text.

Students, regardless of treatment, scored significantly higher on posttest than pretest on both function construction and variable conceptualization. These results may have been influenced by instructional strategies used in the treatments including: inquiry mathematical thinking, schema training, linked representational systems, and the coordinate graph tutorial teaching.

Students receiving instructions via the inductive table-and-graph program scored significantly higher on function construction section of the posttest than did students receiving the table-only treatment. This result is consistent with prepositions recognizing the conceptual richness of visuals, specifically the coordinate graph, in mathematics education learning.

Results from the present study suggest the use of inductive multimedia programs treatments that include many strategies including inquiry learning from data, tutorial, schema, and core representational systems for the problem of translation, specifically creation of linear function. The data specifically suggest that the inductive multimedia programs that include the coordinate graph tutorial strategy in their construction have a significant effect on the function construction tasks.

Introduction

National reports (Dossey, Mullis, Lindquist, & Chambers, 1988; Mullis, 1994; Mullis, Dossey, Owen, & Philips 1993) indicate that many students do not understand mathematical concepts and skills taught in schools. Mathematics proficiency is crucial for the individual pursuing higher education (National Research Council, 1989) and critical to the creation of an informed citizenry and economically competitive society (Anderson et al., 1994).

This lack of mathematics understanding, in algebra for example, forces students to memorize algebraic rules and procedures. Therefore, many students think algebra is simply rule-based memorization (Brown et al. 1988; Kieran, 1992). As a result, students are often unable to apply basic algebraic and geometric concepts to problem solving (Brown et al.). A number of studies indicate that, even in ordinary relational word problems, students have major difficulty generating equations that represent relationships (English & Warren, 1995; Chaiklin, 1989; Clement, 1982; Herscovics, 1989; Lewis & Mayer, 1987; MacGregor & Stacy, 1993; Mayer, 1982).

Researchers have pointed out that instruction in generating linear function needs an approach that considers students' general reasoning processes (English & Warren, 1994) and accounts for conceptual errors in problem translation (Kaput & Simes-Knight, 1983). In addition, instruction must include linked multimedia presentation, emphasizing visual algebraic process representations, including tables, patterns, and graphs (Kaput, 1992; Kosslyn, 1980). To address the difficulty students have in learning how to generate linear functions (or equations) from relational word problems, the following offers a two-part solution. Using linked representational media (multimedia programs), mathematics Computer-Based Instruction (CBI) should stress (1) inductive problem-solving strategies or scientific heuristics (for example, by working backwards, working inductively, or applying algebraic thinking to data) (DeMarois, McGowen, & Whitkanack, 1996; Polya, 1954) and, most importantly, (2) the teaching of the language of mathematics and math visuals (graphs) (Esty, 1992; Kaput, 1992; Bell & Janvier, 1981). Based on an inductive multimedia program developed according to the above approach, this study sought to determine how visuals (table and graph) affect students' creation of linear function construction and variable conceptualization from relational word problems.

Justification

In today's work environment, the ability to think critically, to communicate mathematical ideas, and to develop problem-solving strategies is essential (Smith, 1994). Furthermore, while building toward a career, research indicates that a strong relationship exists between mathematical skills and success in college, regardless of major (Waits & Demana, 1988). However, despite the proven short- and long-term value of math skills, student underpreparedness in mathematics is a continuing and growing problem in higher education (Berenson, Best, Stiff, & Wasik, 1990). In algebraic problem-solving situations, students find algebraic applications difficult. Collegiate mathematics education research (Clement, 1982; Lewis & Mayer, 1987; Lochhead & Mestre, 1988; Mayer, 1982; Wollman, 1983) indicates that most students cannot, in fact, translate rational word problems into simple linear functions.

Several researchers (Bishop, 1989; Clements, 1982; Janvier; 1987) have encouraged research in the exploration, investigation, and curriculum implications of graphical and tabular representations of knowledge. According to Dugdale, Thompson, Harvey, Demana, Waits, Kieran, McConnell, and Christmas (1995), a graphical representation "can reveal insights into the problem situation that are not readily revealed by symbol manipulation alone" (p. 330). On the other hand, studies (Carpenter, Corbit, Kepner, Lindquist, & Reys, 1981; Goldenberg, Lewis, & O'Keefe, 1992; Kerslake, 1981; Monk, 1992) indicate that graphical presentation adds its own ambiguity to the learner's syntactic translation problem. Despite the ambiguous role of graphical presentations in problem solving, various researchers support the use of visuals: as mnemonic tools (Atkinson, 1975; Lewis, 1989; Winn, Tian-Zhu, & Schill, 1991), as gestalt-producing mental processors (Skemp, 1989), and as mathematics language (Esty, 1992; Janvier, 1987; Kaput, 1989). Graphs, for example, enable problem solving because a graph allows one to view a single graphical entity instead of a binary quantitative relationship (Kaput, 1989), which is, by definition, a more complex matrix from which to draw a solution.

Kosslyn (1994) has shown that human visual perception and cognition have strengths and limitations, and their measures depend on both the quality of visuals displayed and the adequate usage of those visuals. With their visual (graphic) and authoring programs, low-cost microcomputers have removed many visual cognitive obstacles in mathematics teaching, as in their numerous other applications developed to aid cognition. Computers may help us to use visuals more adequately. For example, graphs can be made even more cognitively engaging via animated displays and linked explanations. Authoring programs facilitate development of materials that allow visual learning as well as inductive reasoning. Computers can be used to develop visuals and support reasoning processes, as well as to measure whether, based on a set of dependent variables, visual treatments differ on average.

Although researchers have not taken up his mandate, Clements (1982), a recognized expert in mathematics education, has concluded that despite the fact that clear guidelines for the use of visuals in classroom practice have not yet emerged, "there should not be a reduction in the amount of research which is aimed at achieving this end" (p. 36). Clements stresses that "Mathematics educators need to develop better instruments for assessing the role of visual imagery in mathematics learning" (p. 36). The classroom application of computer-supported or computer-generated graphs and of graphic language are important and viable areas of research in mathematics teaching (Bell & Janvier, 1981; Bishop, 1989; Drefus & Eisenberg, 1987; Eisenberg & Drefus, 1991; Lesh, 1987). Currently, however, little research exists on (1) combined visual effects, (2) inductive software program training, (3) strengths and limitations regarding the order of visual presentation, or (4) any combination of the above. Since the use of visual thinking in mathematics learning is controversial, multivariate research considering many factors may help clarify the role or roles of visuals in problem solving.

The present study considers a multivariate approach to the role of visuals in problem solving based on the need that exists for inductive algebraic multimedia software programs that strongly support the growing interest in the use of visuals in mathematics-based cognitive processes. The graphical presentation and construction capabilities of software programming currently offer the most practical way of producing and using quality visuals. The mere inclusion of graphs is not enough, however; programs should also address the language of graphs and provide solutions to students' graphical misconceptions in a dialogue with learners. Algebraic software programs should consider graphical cognitive obstacles, graph language, and the order of visual presentations. The present study looks for a strong possible visual effect via the combination of tables and graphs with graph language in an inductive multimedia program to improve linear function construction and conceptualization of variables.

Hypotheses

Hypothesis 1: Students receiving instructions via either software program will score higher on the posttest than on the pretest in both areas: linear function and variable conceptualization.

Hypothesis 2: Students receiving instructions via the inductive table-and-graph program will score higher on the posttest in both areas than will students receiving the table-only treatment.

Subjects

The subjects were 98 undergraduate students enrolled in one of two sections of EMC 321, Computer Literacy, offered by the College of Education during the Fall 1997 semester. Subjects ranged in age from 18 to 25 years.

Treatments

Two inductive multimedia programs served as the instructional treatments for this study. Both self-paced treatments were developed by the author using Authorware 3.0 via Macintosh. Both programs, *InductiveThinker Table* and *InductiveThinker Table & Graph*, had two lessons. Lesson one contained information about the input, output, and independent and dependent variables. Lesson two included information about the rate of change (or the slope of the function) and linear function creation. Some screens were added to *InductiveThinker Table* to construct *InductiveThinker Table & Graph.* The pace was user controlled and subjects had the ability to navigate between pages, sections, or lessons and could exit the program at any time.

The instructional treatments were evaluated via two types of formative evaluation procedures described by Dick and Carey (1985): one-to-one and small-group. After a review by three College of Education instructors, which suggested a major change regarding method, level of control, and content, the treatments were piloted using seven students similar to the target population who were unfamiliar with the programs.

Instruments

Two instruments were used to collect data:

Instrument 1: As one of the primary measures, the first instrument was a 12-item pretest administered to both groups at the beginning of the data collection. Test items were in short-answer form (both English and algebraic) and were parallel to the practice items contained in *InductiveThinker Table*, the first program. The pretest items measured student achievement on the instructional themes specifically stated in the *InductiveThinker Table* program.

Instrument 2: Another of the primary measures, the second instrument was a 12-item posttest administered to both groups at the end of the treatment. Test items were again in short-answer form (both English and algebraic) and were parallel to both the items contained in the pretest and to the practice items contained in *InductiveThinker Table*, the first program. The first six questions of both the pretest and the posttest paralleled the classic student-professor example, to some extent. The last six items of both the pretest and the posttest were related to the conceptualization of variables.

Procedures

Subjects were randomly assigned to two treatment groups. Both treatment programs were delivered via Macintosh computers in Farmer 214, a computer lab in the College of Education. Throughout the self-administration of the program, subjects spent as much time as they desired on any portion of the program. Subjects from each group viewed the treatments individually, under the same physical conditions, and all received extra credit for participating in the study. Subjects chose their own participation times from a variety of times offered and were evaluated via a pretest before the treatment and a posttest upon completion of the programs.

Design

This study used a Pretest-Posttest Control Group design. A pretest of the dependent variable was administered to both groups. Then, the control group received an inductive table treatment, while the experimental group received a manipulated treatment (Inductive Table and Graph). Finally, both groups were posttested. The treatment forms were A versus A+B.

Validity of the Research Design

The Pretest-Posttest Control Group design was selected because the subjects had a variety of mathematical backgrounds and came from different disciplines. The combination of (1) random assignment, (2) the presence of a pretest, (3) the presence of a control group, and (4) the short data collection period controls for threats to internal validity, such as statistical regression, mortality, maturation, history, testing, and instrumentation.

Statistical Analysis

Multiple Analysis of Covariance (MANCOVA) was used to determine whether there existed a significant difference between the posttest scores of subjects who received Inductive Table instruction and those who received Inductive Table and Graph instruction.

Limitations

The sample for this study was selected from two sections of an undergraduate course offered by the College of Education at Arizona State University. The three-credit course was offered during the Fall 1997 semester. Selection was based on the willingness of instructors to facilitate the study and students to participate in the study. In addition, only undergraduates enrolled for credit (rather than audit) were selected. Subjects were randomly assigned to one of two groups.

Significance of the Study

Significantly higher test scores from those students who received the Inductive Table and Graph instruction (1) would lead to a useful guideline for computer-based classroom and at-a-distance algebraic practices and (2) would provide positive support that, when they are taught and used properly, graphical presentations offer a powerful holistic visual aid to the complex abstraction of mathematics. In addition, this study may introduce support for the role of graphs in theories of imagery.

Results

The purpose of this study was to determine the effects of an inductive multimedia program including graphs and tables on subjects' ability to create linear functions and construct variables from word problems. Subjects viewed one of two self-paced inductive multimedia programs. The first program, *InductiveThinker Table*, included tables in addition to other elements (e.g., animation). The second version of the program, *InductiveThinker Table & Graph*, included both tables and graphs. This program was identical to the first version in all respects except for the addition of the graph visuals and graph-related text.

Demographic Data

The data collection began with 101 undergraduate students enrolled in two sections of Computer Literacy, in a large South western university. The scores of three students were discarded because they did not finish the posttest. Of the remaining 98 students, 69 were females and 29 were males (70% and 30% respectively) with mean age of 22.6 years for both males and females. Thirty-six percent of students reported majors in Communication or Journalism, 23% in education, 10% in justice studies, and 31% others.

Subjects were randomly assigned to the treatments and the size of the sample was not fixed, resulting in groups of different sizes. The size of the sample for treatment 1, *Inductive Thinker Table*, was 47 subjects and for treatment 2, *Inductive Thinker Table and Graph*, 51 subjects.

A split-half (odd-even) reliability test was conducted on the 12 items pretest and posttest to assess linear function construction and variable conceptualization. Coefficient alphas were computed to obtain internal consistency estimates of reliability for these two scales. The alphas for the pretest and posttest scales were .70 and .61 respectively. The tests measured the intended content areas: Function construction and variable conceptualization. Pretest and posttest items were parallel to the practice items contained within the instructional programs and were short-answer and multiple choice in format.

Subjects were schedule to spent one hour on treatments. However, the lab observations indicated that subjects spent less than one hour with the programs. Subjects often came late and rushed through to finish the programs in order to catch their next class. Most students came during the lab rush-hour, often during times when they were supposed to be in their computer literacy class. Many subjects were bored and tired at the end of the treatments.

Data Analysis

Two hypotheses were considered. The first involved examining whether the mean difference between pretest and posttest scores on two levels (function construction and variable conceptualization) were significantly different from zero. One-way, repeated-measure, analysis of variance (ANOVA) was used to measure mean differences between pretest and posttest scores, first, of function construction and second, of variable conceptualization.

The second hypothesis involved examining whether the posttest adjusted means on a set of dependent variables (function creation and variable conceptualization) varied significantly across the two factors (Table and Table & Graph treatments). A multivariate analysis of covariance (MANCOVA) was used to determine whether groups differed on the dependent variables. The Statistical Package for the Social Sciences (SPSS) was selected for use in this study. An alpha level of .05 was used for all statistical tests.

Hypothesis 1

Students receiving instructions via either software program will score higher on the posttest than on the pretest in both areas: linear function and variable conceptualization.

A one-way repeated-measure ANOVA was conducted with the factor being either treatment and the dependent variable being the pretest and posttest scores for function construction. The means, highest possible scores, number of subjects, and standard deviations of scores are presented in Table 1. The results for the ANOVA showed a significant treatment effect on function construction regardless of kind of effect, Wliks's Lambda = .457, $F(1, 97) = 115.27$, $p < .001$, multivariate d (Eta Squared) = .543.

Table 1. Means, Highest Possible Score, Number of Subjects, and Standard Deviations for Function Construction Scores

Dependent Variable	M	Highest Score	N	SD
Pretest	1.97	6.00	98	1.79
Posttest	4.11	6.00	98	1.52

A follow-up pairwise comparison, univariate test was conducted. The results confirmed the Multivariate test indicating that the posttest mean ($M = 4.11$, SD = 1.52) was significantly greater than the pretest mean (M 1.97, $SD = 1.79$), $t(97) = 10.74$, $p < 0.008$.

Another one-way repeated-measure ANOVA was conducted with the factor being treatment and the dependent variable this time being the pretest and posttest scores for variable conceptualization. The means, highest possible score, number of subjects, and standard deviations of scores are presented in Table 2. The results for the ANOVA showed a significant treatment effect on variable conceptualization regardless of kind of effect, Wliks's Lambda = .622, $F(1, 97) = 58.84$, $p < .001$, multivariate d (Eta Squared) = .378.

Table 2. Means, Highest Score, Number of Subjects, and Standard Deviations for Variable Conceptualization Scores

Dependent Variable	M	Highest Score	N	SD
Pretest	4.06	6.00	98	1.21
Posttest	5.01	6.00	98	0.83

A follow-up pairwise comparison, univariate test was conducted. The results confirmed the Multivariate test indicating that the posttest mean ($M = 5.01$, SD = .83) was significantly greater than the pretest mean ($M = 4.06$, $SD = 1.21$), $t(97) = 7.68$, $p < .001$.

Hypothesis 2

Students receiving instructions via the inductive table-and-graph program will score higher on the posttest in both areas than will students receiving the table-only treatment.

A MANCOVA with two dependent variables and two covariates was conducted. The independent variable, instructional treatment, included two levels: inductive table and inductive table and graph programs. The dependent variable, posttest scores, also included two levels: function construction and variable conceptualization. The covariates were pretest scores on both function construction and variable conceptualizations.

The MANCOVA indicated that the adjusted population mean vectors (posttest scores) were significantly different among the groups at the .05 level ($F = 4.77$, $p < .001$). The MANCOVA's first assumption (check to see that there is significance relationship between the dependent variables and the covariates) was verified. Sample size justified using two covariates ($C < 8$, where C is the number of covariates). Tables 3 and 4 show the means and adjusted means for treatments of the set of dependent variables and the means of pretests.

Table 3. Means of posttests and pretests for Function Construction

Treatment	Observed M	Adjusted M
Inductive Table (Posttest)	3.70	3.69
Inductive Table & Graph (Posttest)	4.50	4.51
Inductive Table (Pretest)	1.99	-----
Inductive Table & Graph (Pretest)	1.97	-----

Table 4. Means of posttests and pretests for Variable Conceptualization

Treatment	Observed M	Adjusted M
Inductive Table (Posttest)	4.83	4.89
Inductive Table & Graph (Posttest)	5.12	5.12
Inductive Table (Pretest)	4.03	-----
Inductive Table & Graph (Pretest)	4.67	-----

A separate analysis of covariance was done on each dependent variable. The probability indicated that only the function construction variable was significant at the level .05 (Table 5). The power on the variable conceptualization was .31. Probability of students receiving instructions via either treatment and score significantly different on the posttest in variable conceptualization is very low. Low power indicates a low probability of rejecting the null hypothesis.

Table 5. Analysis of Covariance on Dependent Variables

VARIABLE	MS	F	p	Power
Function Creation	16.38	8.3	.005	.81
Variable Conceptualization	1.33	2.2	.139	.31

The multivariate test for the homogeneity of the regression hyperplanes was not significant at the .05 level ($F = .237$, $p < .917$) indicating that the assumption of homogeneity was quite tenable. The multivariate F, corresponding to Wilk's lambda, indicated that there is a significant difference between the set of dependent and the set of covariates at the .05 level ($F = 6.367$, $p < .001$). Table 6 indicates that 22.83% of the within variability on variable function construction is accounted for by two covariates, pretest function construction and variable conceptualization.

Table 6. Univariate Tests for Relationship Between Dependent Variables and Covariates

VARIABLE	SS	MS	F	P
Function Creation	22.83	11.42	5.8	.004
Variable Conceptualization	8.68	4.34	7.3	.001

The Bryant-Paulson procedure was conducted because of the possibility of measurement error on the covariant of low or questionable reliability. The question was whether there was a significant difference between the adjusted means on function construction for the groups. Results again indicated a significant difference, Hotelling Value = .0001, interpolated critical value = 2.83, BP = 5.78.

Discussion

Hypothesis 1: Overall Treatment Effects

The results of the current study support Hypothesis 1. Students, regardless of group or treatment, scored significantly higher on posttest than pretest on both function construction and variable conceptualization.

This improvement is not likely due to learning during pretest. The pretest did not measure factual information which could be recalled. Rather, the translation problem is a cognitive obsticle. Subjects needed an effective treatment that addressed conceptual understanding. "Taking a pretest on algebraic equations . . . is much less likely to improve performance on a similar posttest" (Gay, 1992, p. 304).

The results of this study are consistent with the finding of Wollman's (1983) study. Using a tutorial strategy, Wollman found that 94 percent of his subjects constructed the correct equation. However, later, Wollman (1985) agreed that his subjects could have used the checking procedure to construct equations regardless of conceptual understanding. Observations conducted during the current study revealed that students did not check their answers during the tests and used either table or graphical procedures using arbitrary or given data to create their equations.

Results also indicated that students did significantly better on the variable conceptualization portion of the posttest. Students' success with variable conceptualization may have positively impacted their success with function construction. If this is the case, then Clement's (1982) report that variable understanding is one of the key issues in successful problem solution is supported by the current study.

The results might have been influenced by the combination of other instructional strategies represented by the current study's instructional treatments. These instructional strategies include learning from (1) mathematical thinking, (2) schema training , (3) linked representational systems, and (4) the coordinate graph.

The effects of mathematical thinking required by the current study's programs might have caused the improved results of the present study. Shoenfeld's (1992) mathematical thinking is embedded in engagement in scientific research, the science of patterns, and the determination of regularities in systems. The methodological framework of the *InductiveThinker* treatments included these attributes. The treatments of the current study include the inductive methodology of constructivist epistemology for reasoning and discovering via the construction of tables of variable values.

There might be an interaction effect between the improved results of the current study and the schema-training nature of the treatments. The current study's treatments include schema acquisition that eventually provides rule automation and strengthens metacognitional skills. By using tables of values and describing steps of procedures, for example, treatments increase memory demand that could hinder student's rearrangement of information and ability to construct equations.

The results could also be attributed to the anchored linked-representational systems (Kaput, 1992) property of the treatments. The current study's treatments use an inductive tutorial mode to propose, measure, and evaluate working hypotheses by representing linked representations of mathematical notations, including table, verbal, and graph representations. The programs ask learners to write down their thinking and working models, then immediately compare what they have written with appropriate examples. Students receive tabular, graphical, and numerical feedback instantly during mindful engagement with the treatment programs. In addition to the linked-representational effect, other media attributes such as mind-machine collaboration could be a positive attribute.

Hypothesis 2: Inductive Table versus Inductive Table and Graph

The significant difference is not likely the result of the longer treatment. The inductive table-and-graph program, Treatment 2, included the coordinate graph strategy in addition to other strategies. By teaching the language of graph, the coordinate graph strategy used its attributes, including geometrical graphical representation to further subjects' conceptual understanding of the translational task. Students' difficulty with translational tasks results from their use of natural language syntax and their lack of conceptual understanding of variables and function construction, not from the amount of instruction received. Persistence of translational problems has been detected among freshmen engineering students (Clement, 1982) who received extended mathematics instructions. The difference is likely to be the result of the alternative and graphical core representation of the coordinate graph.

The multivariate results related to the second hypothesis of this study indicating that graph has a significant effect in function construction are consistent with the findings of Schwarz et al. (1990), Tall and Thomas (1989), and Yerushalmy (1988).

The most important outcome of the current study is the second finding, that students receiving instructions via the inductive table-and-graph program scored significantly higher on function construction of the posttest than did students receiving the table-only treatment. Treatment 2, *InductiveThinker Table and Graph*, includes all elements of the first treatment as well as graphical representation and the teaching of the language of the graph elements. The second finding is attributed to the graphic strategy of Treatment 2 and is most likely related to the teaching of the graphic language by the treatment program. Representing linked presentations, including graphics alone, is not likely to be a major factor, since much research (Goldenberg, Lewis, & O'Keefe, 1992; Kerslake, 1981; Monk, 1992) indicates that graphic mediation has its own ambiguity that adds to the learner's syntactic translation problem.

Contrary to expectation, investigation of the second hypothesis revealed that the groups were not significantly different regarding variable conceptualization. One possible explanation for this finding is the length of Treatment 2, which was about half an hour longer, and the fact that the last questions of the posttest were related to variable conceptualization. Observations during data collection revealed that most students rushed through answering the posttest questions; many came late and, after the posttest, had to leave quickly to attend classes across campus. Another explanation might be the fact that, according to a few observation notes, posttest and pretest questions were very similar and students may have thought they were practically the same questions. The result could also be attributed to the possible testing and pretest-treatment interaction effect. A careful reexamination of the measures indicated that tables that were used in pretests for explanation of the variable questions were not used in the function construction section. This may have caused students to score higher on the pretest.

Another possible explanation is that the tests might not have been extensive enough to measure variable conceptualization. Figure 2 (in Chapter 4) indicates a ceiling effect on posttest scores. Majority of subjects achieve the highest scores possible.

Implications of the Study

Results from the present study suggest that for the problem of translation, schools may find it most beneficial to use treatments similar to *InductiveThinker Table and Graph*, that employ all learning strategies, including inquiry learning from data, tutorial, schema, and core representational systems. The present study indicates that the coordinate graph strategy is an important representational system and is very effective in translational tasks only when its language is taught by software programs and that language is understood by learners. Otherwise, the implication is that the use of coordinate graphs simply adds to the learner's syntactic translational problem. Difficulties regarding graph integration, construction, and translation must be recognized and taught to learners before they use graphs.

Before students learn how to construct functions using any of the proposed instructional solutions to the problem of translation, they need to understand the concept and use of variables (Usiskin, 1988). This study did not find significant differences between groups in the area of variable conceptualization; subjects earned maximum scores regardless of the group to which they belonged. However, function construction includes variable recognition. Students must learn to conceptualize variables in order to understand functions.

Strengths and Limitations

This study considered one of the most important problems in the teaching and learning of college algebra: the translation task in word problems. This study indicated that translation from problem situation to symbolic algebra is difficult for students, and that linking reality and mathematics symbolism requires students to engage in various stages of abstraction that include interpretation, construction, and translation. This study combined instruction treatment types examined and recommended by the experts and investigated their combined effects. Based on the literature review, this study used the conceptual richness of visuals in mathematics education — specifically, the coordinate graph, a powerful representational introduction to the complex abstractions of mathematics.

The current study addresses a solution to the problem of student's translation in algebra by proposing and evaluating a solution to the problem of function construction. This study introduced a combined instructional strategy solution that includes the coordinate graph. The combined strategy was measured against another combined strategy without the coordinate graph. This measurement allowed for a more meaningful result than would the comparison of two treatments reflecting disparate strategies. The current study used a gestalt-producing (Kaput, 1989) combined approach by including the coordinate graph in the study's proposed multiple strategies. A graph allows consolidation of pairs of related numbers into points: graphing connects algebra to geometry. A strength of the study was that, in keeping with hypotheses 2, all the effects but visual effect (the coordinate graph) remained constant in order to measure the graphical effect. For both hypotheses, the study used univariate as well as multivariate analysis.

Limitations of this study that may affect validity include (1) subjects' voluntary participation (2) the content of test measures regarding the conceptualization of variable, and (3) limited qualitative observation and analysis. Although the test items regarding the conceptualization of variables were parallel to the exercise items in the programs, more challenging conceptual understanding should be require on the tests. For example, instead of providing multiple-choice questions regarding the conceptual understanding, short-answer responses should be designed. More through qualitative observation and analysis would indicate the particular strategies students used in performing translational tasks.

Recommendations for Future Research

Further research should include the following:

- Expanded Designs: Designs that include a third group that is not exposed to a treatment to reveal possible pretest interaction
- Factorial Designs: Designs which consider the joint effect (joint interactions) of the treatments
- Qualitative Designs: Designs in which students' mathematical (translational) thinking is observed, and in which subjects are interviewed regarding their experience with the treatments

Conduct experimental research that considers other independent factors beside treatments, including sex, IQ, and mathematical background. Such research should measure the possible joint interactions of these factors on the study's output. The strength of this design would be the examination of a joint effect of sex and treatment on the dependent variable(s).Regarding function conceptualization, Drefus and Eisenberg (1983) said, "The challenge is clear; the problem is well defined. We must teach so that our students will be able to grasp global notions and find inter-relationships." Still, a decade later, MacGregor and Stacey (1993) reported that "One of the greatest difficulties for beginners in algebra is linking a mathematical situations to its formal description." Students use a naturalistic representational system — the English language — to translate word problems to functions instead of using a variety of mathematics representational systems that includes the coordinate graph. As shown by the current study, using a combined treatment that includes the coordinate graph, educators can help students to overcome the difficulty or misconception of the translational tasks.

References

Anderson, B. L., Varanka-Martin, M. A., Romagnano, L., Bielenberg, J., Flory, M., Mieras, B., & Whitworth, J. (1994). Issues of curriculum reform in science, mathematics and higher order thinking across the disciplines. Washington, DC: U.S. Office of Research, Department of Education.

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. For the Learning of Mathematics, 14(3), 24-35.

Atkinson, R. (1975). Mnemotechnics in second-language learning. American Psychologist, 30, 821-828.

Authorware [Software]. (1989). Minneapolis, MN: Authorware, Inc.

Barclay, T. (1985). Guess My Rule [Software]. Pleasantville, NY: HRM Software.

Behr, M., & Bright, G. (1983). Mathematics learning in college and beyond. Focus on Learning Problems in Mathematics, 5(3&4), 1-13.

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. For the Learning of Mathematics, 22(1), 34-42.

Bennett, A., & Maier, E. (1996). A visual approach to solving problems. The Mathematics Teacher, 89(2), 108-111.

Berenson, S., Best, M., Stiff, L., & Wasik, J. (1990). Levels of thinking and success in college developmental algebra. Focus on Learning Problems in Mathematics, 12(1), 3-13.

Bishop, A. J. (1980). Spatial abilities and mathematics: A review. Educational Studies in Mathematics, 11, 257-269

Bishop, A. J. (1983). Space and geometry. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 176-203). New York: Academic Press.

Bishop, A. J. (1986). What are obstacles to learning geometry? Studies in Mathematics Education (UNESCO), 5, 141-159.

Bishop, A. J. (1989). Review of research on visualization in mathematics education. Focus on Learning Problems in Mathematics, 11(1), 7-15.

Brown, C. A., Carpenter, T. P., Kouba, V. L., Lindquist, M. M., Silver, E. A., & Swafford, J. O. (1988). Secondary school results for the fourth NAEP mathematics assessment: Algebra, geometry, mathematical methods, and attitudes. Mathematics Teacher, 81, 337-347.

Carlson, M. (in press). What do high-performing college algebra students know about function? Arizona State University.

Carpenter, T. P., Corbit, M. K., Kepner, H. S., Jr., Lindquist, M. M., & Reys, R. E. (1981). Results from the second mathematics assessment of the National Assessment of Educational Progress. Reston, VA: National Council of Teachers of Mathematics.

Chaiklin, S. (1989). Cognitive studies of algebra problem solving and learning. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 93-114). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.

Clement, D., & Sarama, J. (1997). Computers support algebraic thinking. Teaching Children Mathematics, 3(6), 320-325.

Clement, J. (1982). Algebra word problems solutions: Thought processes underlying a common misconception. Journal for Research in Mathematics Education, 13(1), 16-30.

Clement, J., Lochhead, J., & Monk, G. (1981). Translation difficulties in learning mathematics. American Mathematical Monthly, 88, 286-290.

Clement, J., Narode, R., & Rosnick, P. (1981). Intuitive misconceptions in algebra as a source of math anxiety. Focus on Learning Problems in Mathematics, 3(4), 36-45.

Clements, K. (1982). Visual imagery and school mathematics. For the Learning of Mathematics, 2(3), 33-38.

Cornet, L. M. (1986). Improving student preparation. Higher education and the schools working together. Atlanta, GA: Southern Regional Education Board.
(ERIC Document Reproduction Service No. ED 363 272)

DeMarois, P., McGowen, M., & Whitkanack, D. (1996). Applying algebraic thinking to data. New York, NY: Harper Collins College Publishers.

Dick, W., & Carey, L. (1985). The systematic design of instruction (2nd ed.). Glenview, IL: Scott, Foresman.

Dossey, J., Mullis, I., Lindquist, m., & Chambers, D. (1988). The mathematics report card: Are we measuring up? Trends and achievement based on the 1986 National assessment. Princeton, NJ: Educational Testing Service.

Dossey, J. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), Handbook of research on mathematics and learning (pp. 39-64). New York: Macmillan.

Dugdale, S. (1994). Using student's mathematical inventiveness as a foundation for software design: Toward a tempered constructivism. Educational Technology Research and Development, 42(1), 57-73.

Dugdale, S., & Kibbey, D. (1986). Green globs and graphing equation [Software]. Pleasantville, NY: Sunburst Communications.

Dugdale, S., Kibbey, D., & Wagner, L. J. (1991). Slalom, ZOT, ZigZag: Challenges in graphing equations [Software]. Pleasantville, NY: Sunburst Communications.

Dugdale, S., Thompson, P., Harvey, W., Demana, F., Waits, B., Kieran, C., Mcconnell, J., and Christmas, P. (1995). Technology and algebra curriculum reform: Current issues, potential directions, and research questions. Journal of Computers in Mathematics and Science Teaching, 14(3), 325-357.

English, L., & Warren, E. (1995). General reasoning processes and elementary algebraic understanding: Instructions for initial instruction. Focus on Learning Problems in Mathematics, 17(4), 1-19.

Esty, W. (1992). Language concepts of mathematics. Focus on Learning Problems in Mathematics, 14(4), 31-54.

Fey, J. (1989). School algebra for the year 2000. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 93-114). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.

Goldenberg, P., Lewis, P., & O'Keefe, J. (1992). Dynamic representation and the development of a process understanding of function. The function concept: Aspects of epistemology and pedagogy. Washington, DC: Mathematical Association of America.

Hancock, C. (1990). Data modeling with the TableTop. (Technical Report). Cambridge, MA: Technological Educational Resource Centers (TERC).

Hancock, C., & Kaput, J. (1990). Computerized tools and the process of modeling. In G. Booker, P. Cobb, & T. Mendicuti (Eds.), Processing of the 14th annual Meeting of the PME-NA. Oaxtepec, Mexico.

Hancock, C., Kaput, J., & Goldsmith, L. (1992). Authentic inquiry with data: Critical barriers to classroom implementation. Educational Psychologist, 27(3), 337-364.

Healy, S., Hoyles, C., & Sutherland, R. (1990). Peer group discussion in mathematical environments: Final report to the leverhulme trust. Institute of Education, University of London.

Heid, k. (1988). Computer-generated tables: Tools for concept development in elementary algebra. In A. Coxford and A. Schulte (Eds.), The ideas of algebra, K-12 (pp. 170-177). Reston, VA: National Council of Teachers of Mathematics.

Heid, k. (1990). Uses of technology in prealgebra and beginning algebra. Mathematics Teacher, 84, 194-198.

Henle, M. (1997). Modern geometries: The analytic approach. Saddle river, NJ: Prentice-Hall, Inc.

Herbert, K., & Brown, R. (1997). Patterns as tools for algebraic reasoning. Teaching Children Mathematics, 3(6), 340-345.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 60-86). Reston, VA: National Council of Teachers of Mathematics.

Hinsley, D., Hayes, J., & Simon, H. (1977). From words to equation: Meaning and representation in algebra word problems. In M. Just & P. Carpenter (Eds.), Cognitive processes in comprehension. New York: Wiley.

Hoyles, C., & Sutherland, R. (1989). Logo mathematics in the classroom. London: Routledge.

Hutchinson, M. W. (1972). Geometry: An intuitive approach. Columbus, Ohio: Charles E. Merrill.

HyperCard [Software]. (1991). Cupertino, CA: Apple Computer, Inc.

Janvier, C. (1981). Use of situations in mathematics education. Educational Studies in Mathematics, 12, 113-122.

Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), Problems of representation in mathematics and problem solving (pp. 27-31). Hillsdale, NJ: Lawrence Erlbaum Associates.

Johari, Abbas (1997). The InductiveThinker Algebra [Computer software]. Tempe, AZ: Author.

Kaput, J. (1989). Linking representations in the symbol system of algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 60-86). Reston, VA: National Council of Teachers of Mathematics.

Kaput, J. (1992a). Patterns in student's' formalization of quantitative patterns. The function concept: Aspects of epistemology and pedagogy. Washington, DC: Mathematical Association of America.

Kaput, J. (1992b). Technology and mathematics. In D. A. Grouws (Ed.), Handbook of research on mathematics and learning (pp. 515-556). New York: Macmillan.

Kaput, J., & Sims-Knight, J. (1983). Errors in translations to algebraic equations: Roots and implications. Focus on Learning Problems in Mathematics, 5(3&4), 63-78.

Kaur, B., & Boey Peng, B. (1994). Algebraic misconceptions of first year college students. Focus on Learning Problems in Mathematics, 16(4), 43-58.

Kerslake, D. (1981). Graphs. In K. Hart (Ed.), Children's understanding of mathematics: 11-16. London: John Murray.

Kieran, K. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), Handbook of Research on Mathematics and Learning (pp. 390-419). New York: Macmillan.

Kosslyn, S. (1994). Elements of graph design. New York: Freeman Press.

Kosslyn, S., & Pomerantz, J. (1977). Imagery, propositions, and the form of internal representations. Cognitive psychology, 9, 52-76.

Kosslyn, S. (1980). Image and mind. Cambridge (Massachusetts): Harvard University Press.

Kosslyn, S. (1989). Understanding charts and graphs. Applied Cognitive Psychology, 3, 185-225.

Kosslyn, S. (1992). Wet mind: The new cognitive neuroscience. New York: Free Press.

Kozma, R. B. (1991). Learning with media. Review of Educational Research, 61(2), 179-212.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Function, graphs, and graphing: Tasks, learning, and teaching. Review of Educational Research, 60, 1-64.

Lewis, A. (1989). Training students to represent arithmetic word problems. Journal of Educational Psychology, 81(4), 521-531.

Lewis, A., & Mayer, R. (1987). Student's misconceptions of relational statements in arithmetic word problems. Journal of Educational Psychology, 79(4), 363-371.

Lochhead, J. & Mestre, J. (1988). From words to algebra: Mending misconceptions. In A. Coxford and A. Schulte (Eds.), The ideas of algebra, K-12 (pp. 127-135). Reston, VA: National Council of Teachers of Mathematics.

MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. Journal for Research in Mathematics Education, 24(3), 217-232.

Mansfield, H. (1985) Points, lines, and their representations. For the Learning of Mathematics, 5(3), 2-6.

Mayer, R. (1982). Memory for algebra story problems. Journal of Educational Psychology, 74(2), 199-216.

Mayer, R. (1992). Cognition and instruction: Their historic meeting within educational psychology. Journal of Educational Psychology, 84(4), 405-412.

Mullis, Ed. (1994). American's mathematics problem: Raising student achievement (Report No. 23-FR-03). Washington, DC: U.S. Department of Education.

Mullis, I., Dossey, J. A., Owen, E. H., & Philips, G. W. (1993). Executive summary of the NAEP 1992 mathematics report card for nation and states (Report No. 23-ST03). Washington, DC: U.S. Department of Education.

Monk, S. (1992). Student's understanding of a function given by a physical model. The function concept: Aspects of epistemology and pedagogy. Washington, DC: Mathematical Association of America.

Morelli, L. (1992). A visual approach to algebra concepts. The Mathematics Teacher, 85(6), 434-437.

National Council of Teachers of mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Research Council. (1989). Everybody Counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Novick, L., & Holyoak, K. (1991). Mathematical problem solving by analogy. Journal of Experimental Psychology: Learning, memory, and Cognition, 17, 398-415.

Pedoe, D. (1978). Geometry and the liberal arts. New York, NY: St. Martin's Press.

Philipp, R. A. (1992). The many uses of algebraic variables. The Mathematics Teacher, 85(7), 557-561.

Polya, G. (1957). How to solve it (2nd ed.). Princeton, NJ: Princeton University Press.

Polya, G. (1954). Mathematics and plausible reasoning: Induction and analogy in mathematics. Princeton, NJ: Princeton University Press.

Polya, G. (1954). Mathematics and plausible reasoning: Patterns of plausible inferences. Princeton, NJ: Princeton University Press.

Presmeg, N. (1986). Visualization in high school mathematics. For the Learning of Mathematics, 6(3), 42-46.

Richards, J. (1991b). Mathematics discussions. In E. von Glasersfeld (Ed.), Radical constructivism in mathematics education. Dordrecht, Netherlands: Kluwer.

Richards, J., Feurzeig, W., & Carter, R. (1991a). The Algebra Workbench [Software]. Cambridge, MA: BBN Labs.

Rosnick, P., & Clement, J. (1980). Learning without understanding: The effect of tutoring strategies on algebra misconceptions. Journal of Mathematics Behavior, 3(1), 3-27.

Schoenfeld. A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), Handbook of Research on Mathematics and Learning (pp. 334-370). New York: Macmillan.

Skemp, (1987). The psychology of learning mathematics. Hillsdale, NJ: Lawrence Erlbaum.

Soloway, E., Lochhead, J., & Clement, J. (1982). Does computer programming enhance problem solving ability? Some positive evidence on algebra word problems. In R. J. Seidel, R. E. Anderson, & B. Hunter (Eds.), Computer literacy. New York, NY: Academic Press.

Sutherland, R. (1991). Some unanswered research questions on the teaching and learning of algebra. For the Learning of Mathematics, 11(3), 40-46.

Thompson, P. (1989). Artificial intelligence, advanced technology, and learning and teaching algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 60-86). Reston, VA: National Council of Teachers of Mathematics.

Tarte, L. (1990). Spatial orientation skill and mathematical problem solving. Journal of Research in Mathematics Education, 21, 216-229.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. Coxford and A. Schulte (Eds.), The ideas of algebra, K-12 (pp. 8-19). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1997). Doing algebra in grades K-4. Teaching Children Mathematics, 3(6), 346-356.

Wagner, S. (1981). Conservation of equation and function under transformations of variable. Journal for Research in Mathematics Education, 12, 107-118.

Waits, B., & Demana, F. (1988). Relationship between mathematics skills of entering students and their success in college. The School Counselor, 34(4), 307-310.

Wenger, R. (1987). Cognitive science and algebra learning. In A. H. Schoenfeld (Ed.), Cognitive science and mathematics education. Hillsdale, NJ: Erlbaum.

Winn, W., Tian-Zhu, L., & Schill, D. (1991). Diagrams as aids to problem solving: Their role in facilitating search and computation. Educational Technology Research and Development, 39(1), 17-29.

Wollman, W. (1983). Determining the sources of error in a translation from sentence to equation. Journal of Research in Mathematics Education, 14(3), 169-1.

NOTICE

REPRODUCTION BASIS

☒ This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

☐ This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").